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Quantum Effects, Inflationary Model, and Observable Universe

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The very early stages of evolution of the universe might have a profound implications for its further evolution and presently observed large scale structure. It is shown that if the universe passed through an inflationary phase after the quantum stage then the effects of quantum gravity have been washed out and the presently observed distribution of matter is a consequence of processes occurring during and after the inflationary phase.

It is generally believed that effects of quantum gravity could play a significant role only in very special, extreme situations, e.g., at very early stages of evolution of the universe and close to space-time singularity, which is predicted to form in the process of catastrophic gravitational collapse. From the point of view of nonpractitioner I would like to look at the possible imprints of effects of quantum gravity and grand unified theories on the present structure of the universe.

The standard hot big bang model of the universe is very simple and provides amazingly accurate description of its evolution. In order to discuss some of the problems of the standard model let me briefly review the basic observational facts.

As is well known the universe is expanding. The velocity of recession of a galaxy as measured by an observer on Earth is proportional to its distance. The proportionality coefficient is the Hubble constant. The present value of the Hubble constant is in the range 75 ± 25 km s^{-1} Mpc^{-1}. In the first approximation, disregarding local irregularities, the expansion is isotropic. Counts of galaxies and radio sources indicate that in the large-scale matter in the universe is distributed isotropically and homogeneously.

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The isotropy of the universe on a large scale is very precisely confirmed by observations of the radiation background in X rays, microwave, and radio wavelengths. Observed abundance of primordial light elements indicates, that at the very early stage of evolution, about 1 s past the initial singularity the universe was hot and was expanding isotropically.

Assuming that general relativity is the correct theory of gravity and that the position of Milky Way is typical, and using available observational information we can quite reliably reconstruct the thermal history of the universe and discuss important physical processes starting from an epoch when the universe was filled in with hot quark–gluon plasma (see a recent review by Barrow, 1983). Advances in the program of unifying elementary interactions allow us to consider even earlier epochs much closer to the Planck era, when classical general relativity supposedly breaks down and effects of quantum gravity (supergravity) play a dominant role.

In spite of unquestionable successes there are however observational facts which cannot be explained in the framework of the big bang model. Among the most important on the list are: the large-scale isotropy and homogeneity of the universe, isotropy of the microwave background radiation, close to critical mean energy density, present ratio of photons to baryons, baryon asymmetry, and small-scale distribution of matter in the universe. Statements that the universe is as it is because it was as it was are not very satisfactory, as is the point of view that the initial conditions at the very early stage (the Planck era) were such that the universe evolved into its present state. We would like to understand the physical processes which determined the present structure of the universe. Since a satisfactory, consistent quantum gravity theory has not yet been discovered there is no hope, at the moment, to complete this program. There are however results which are encouraging.

In 1969 Parker and, independently, Zeldovich (1970) pointed out that at the early stage of evolution of the universe due to coupling of quantum fields to time-dependent gravitational field particles could be spontaneously created. It was later shown by Zeldovich and Starobinsky (1971) and confirmed by detailed calculation by Hartle and Hu (1979) that the back-reaction of created particles on initially homogeneous but anisotropic universe can smooth out the anisotropy. This process is very effective and even significant anisotropy present at the Planck time is quickly dissipated on a time scale comparable to the fraction of the Planck time. This process might be even more effective in full quantum theory of gravity. For discussion of main results and technical details see Hartle (1981).

Already in 1967 Sakharov pointed out that the observed asymmetry of baryons and the photon to baryon ratio could be explained in a theory of strong interactions which does not conserve baryon number, and he also noticed that such theory should violate CP. These conditions are satisfied by a large class of grand unified theories of strong, electromagnetic, and weak interactions. Weinberg (1979) has shown that in the framework of grand unified theories it is possible to account for observed asymmetry of baryons and entropy per baryon (see the recent review by Langacker, 1981).

Let us now concentrate on the problem of small-scale distribution of matter in the universe. It is commonly accepted that stars, galaxies, and clusters of galaxies appeared as a consequence of growth of small initial perturbations.

The evolution of small initial perturbations in the homogeneous and isotropic Friedman model was analyzed by Lifshitz in 1946. Lifshitz showed that density perturbations are unstable and in a perfect fluid with equation of state $p=(\gamma-1)\rho$, where ρ is the density, p the pressure, and γ is a constant $1 \leqslant \gamma \leqslant 2$ density perturbations $\delta\rho/\rho$ of a scale λ larger than the size of the horizon are described by

$$\frac{\delta\rho}{\rho} = A_1 t^{\frac{2(3\gamma-2)}{3\gamma}} + A_2 t^{\frac{\gamma-2}{\gamma}} \tag{1}$$

where A_1 and A_2 are the two independent amplitudes, which can depend on position. Amplitude of the growing mode A_1 is connected with variations of spatial curvature and the amplitude of the decreasing mode A_2 is connected with nonsimultaneity of the initial singularity and spatially varying anisotropies in the expansion flow. The density perturbations $\delta\rho/\rho$ of scale λ generate metric perturbations δg

$$\delta g \sim \frac{\delta\rho}{\rho}\left(\frac{\lambda}{ct}\right)^2 \tag{2}$$

If the spectrum of density perturbations is described by $\delta\rho/\rho \sim M^{-\alpha}$ the corresponding metric perturbations are $\delta g \sim M^{2/3-\alpha}$. From this relation it follows that the Friedmanian character of the model can be preserved only if $\alpha = 2/3$. Density perturbations with the spectrum $\delta\rho/\rho \sim M^{2/3}$ generate metric perturbations which do not depend on scale. They are called constant curvature perturbations. Their exceptional properties were noticed and discussed by Harrison (1970) and independently by Zeldovich (1972). Constant curvature perturbations play an important role in the adiabatic theory of galaxy formation. Harrison noticed that constant curvature fluctuations might have been produced by quantum fluctuations at the Planck time.

As early as in 1957 Wheeler, using a simple dimensional argument, showed that quantum fluctuations in the gravitational field could produce metric perturbations δg:

$$\delta g \sim \left(\frac{l_P}{\lambda}\right)^2 \tag{3}$$

where $l_P = (\hbar G/c^3)^{1/2}$ is the Planck length. To relate the metric perturbations at the Planck time $t_P = l_P/c$ with the density perturbations we use the Einstein field equations and so we have

$$\delta g \sim l_P^2 \delta R \sim t_P^2 G \delta\rho \sim \left(\frac{\delta\rho}{\rho}\right)_P \tag{4}$$

Comparing (3) and (4) we have $\delta\rho/\rho \sim M^{-2/3}$. This heuristic result indicates that there might be an intricate relation between quantum gravity and small-scale structure of the universe.

This hope has recently been shaken up by the new model of the early evolution of the universe the so-called inflationary universe. The inflationary model of the early evolution of the universe was proposed by Guth (1981) and was recently improved by Linde (1982) and Albrecht and Steinhardt (1982). Guth (1981) noticed that first-order phase transition predicted by grand unified theories, which occurs when temperature T is of the order of 10^{14} GeV could have profound cosmological consequences. The inflationary scenario requires that the Higgs field effective potential $V(\phi)$ have a global minimum at zero temperature. This global minimum is called the true vacuum. The zero temperature effective potential should also possess a second metastable extremum at $\phi = 0$, which is called the false vacuum. Furthermore one assumes that there is a critical temperature $T_c \simeq 10^{14}$ GeV above which the finite temperature effective potential has a lower value near the false vacuum than it has near the true vacuum. For any small temperature $T > 0$ the false vacuum is stabilized by a bump in the finite-temperature effective potential. The height of this bump is of the order T^4 and its width is of order T.

If at the very early stages of the evolution of the universe there was a hot region ($T > 10^{14}$ GeV), which was expanding fast enough to cool to T_c before gravitational effects could cause it to collapse then it will cool to T_c and supercool. In the supercool phase the constant false vacuum energy density determines the rate of expansion and this region expands exponentially. Evolution of this part of the universe can be described by the de Sitter metric with the false vacuum energy playing the role of cosmological constant. The Higgs field fluctuates around the false vacuum. Some fluctuations could grow around the false vacuum. Some fluctuations could be large enough to start to roll over the potential barrier. If they roll over sufficiently slowly the de Sitter phase could last long enough to allow many e-folding times to pass. This region of space-time (bubble) will grow and become larger than the present observable universe. Finally the Higgs field reaches the steep part of the potential, it quickly rolls down and oscillates about the true vacuum position. These oscillations are effectively damped out through coupling to other fields and the released energy is thermalized. The universe reheats to temperature $T \simeq T_c \simeq 10^{14}$ GeV. The subsequent evolution follows the standard big bang scenario.

If the de Sitter phase could last for at least 60 e-folding times than the horizon would grow and become larger than the presently observed universe. Since after the transition to the true vacuum state the universe is reheated to $T \simeq T_c$ and the vacuum energy is transformed into radiation energy the radiation energy is enhanced by a factor $\simeq 10^{52}$ relative to the curvature term. These properties of the inflationary model are very important. They can be used to explain several, mentioned earlier, cosmological conundrums of the standard model, namely, the isotropy of the microwave background radiation (the horizon problem), the observed large scale isotropy of the universe, the near critical vacuum energy density, and others.

It was shown by Frieman and Will (1982) that the inflationary scenario is stable with respect to small-density perturbations. Hawking and Moss (1982) suggested that there is a cosmological "no hair" theorem and the de Sitter space-time is dynamically stable.

It was also shown that the new inflationary model is compatible with arbitrarily large initial anisotropy (Demianski, 1984). To show that let us consider an anisotropic Bianchi type-I universe filled with radiation and constant false vacuum energy density ρ_v. The evolution of this model is described by

$$\dot{R}^2 = \frac{8\pi G}{3}(\rho_r + \rho_v)R^2 + \frac{8\pi G}{3}\frac{a^2}{R^4} \tag{5}$$

and

$$\rho_r R^4 = \text{const} \tag{6}$$

where R is related to the volume expansion parameter by $\dot{V}/V = 3(\dot{R}/R)$, ρ_r is the radiation energy density, and a is a constant related to the initial anisotropy. Equation (5) can be rewritten in the form

$$\frac{dz}{d\tau} = 2(z^2 + 1 + \beta/z)^{1/2} \tag{7}$$

where

$$\tau = \chi t = \left[\frac{8\pi G}{3}\rho_v\right]^{1/2} t, \qquad z = \left(\frac{R}{R_0}\right)^2 \left(\frac{\rho_v}{\rho_{r(0)}}\right)^{1/2}$$

$$\beta = \frac{a^2}{R_0^6 \rho_{r(0)}} \left[\frac{\rho_v}{\rho_{r(0)}}\right]^{1/2} = \frac{\rho_{an(0)}}{\rho_{r(0)}} \left[\frac{\rho_v}{\rho_{r(0)}}\right]^{1/2}$$

$R_0 = \text{const}$, and $\rho_{r(0)}$ and $\rho_{an(0)}$ are correspondingly the initial energy density of radiation and anisotropy.

In equation (7) z^2 is related to the false vacuum energy density, 1 to the radiation energy density, and β/z to the anisotropic energy density. If $\beta \gg 1$ and the anisotropic energy density determines the expansion rate of the universe $z = \beta^{1/3}(3\tau)^{2/3}$. The radiation energy density becomes comparable with the vacuum energy density when $z = 1$ corresponding to $\tau = (1/3)\beta^{-1/2}$. The vacuum energy density starts to play a dominant role when $z = \beta^{1/3}$ corresponding to $\tau = 1/3$.

The de Sitter phase occurs if the small fluctuations of the Higgs field generated at the moment when the universe cools to T_c ($\rho_v = \rho_r$ or $z = 1$) do not have enough time to significantly grow by the time when the vacuum energy density starts to play a dominant dynamical role.

Let us assume that the initial fluctuations of the homogeneous Higgs field are sufficiently large so that their evolution can be described by the classical evolution equation

$$\ddot{\phi} + 3H\dot{\phi} = \lambda\phi^3 \tag{8}$$

where $\lambda \simeq 1/2$ and H is the Hubble constant. It is easy to check that the approximate solution is

$$\phi = \frac{\phi_i}{\left[1 - \lambda(\phi_i/\chi)^2(\tau - 1/3\sqrt{\beta})^2\right]^{1/2}} \tag{9}$$

Following Guth we assume that the initial fluctuations are of the order of thermal fluctuations and $\phi_i \simeq 0.23\chi$. By the time when the vacuum energy starts to determine the expansion rate of the universe the fluctuations grow only insignificantly $\phi(\tau = 1/3)/\phi_i \simeq 1.01$. Therefore the vacuum energy survives up to the moment when it starts to play a dominant dynamical role and the universe stays in the false vacuum state for many e-folding times.

One might ask what will happen when due to coupling to other fields the anisotropic energy density is dissipated and released in the form of heat.

This will only delay the moment when the phase transition occurs but it will not alter our conclusion.

If it turns out that the inflationary model provides the correct description of the very early evolution of the universe the problem of initial conditions, which we have to specify, would disappear. It would be enough to assume that initially there was a small region of the very early universe which was homogeneously expanding until the moment, when vacuum energy started to play a dynamically important role. In the inflationary model this is the only restriction which we have to impose on initial conditions. During the de Sitter phase this small region will expand to encompass total mass much larger than the mass contained in the presently observable part of the universe. After reheating this homogeneous and isotropic region will be filled with radiation of temperature T_c and its mean energy density will be practically equal to the critical density.

From the analysis of Frieman and Will (1982) it follows that any primordial density perturbations existing in the pre-de-Sitter phase will be washed out. To see if other types of primordial perturbations could survive the inflationary phase, let us consider the purely gravitational perturbations. Since details of the phase transition and the reheating process are not known, let us assume that both processes are instantaneous. We also assume that the universe is homogeneous and isotropic, with Euclidean spacelike sections, and initially is radiation dominated. When the universe cools to certain critical temperature T_c the false vacuum energy density starts to play a dynamically dominant role and the universe expands exponentially. After many e-folding times the latent heat is released and the universe reheats again to T_c and becomes radiation dominated.

Our model is described by the line element of the form

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2) \tag{10}$$

where

$$a(t) = \begin{cases} a_0(t)^{1/2}, & t \leqslant t_1 \\ a_0(t_1)^{1/2} e^{\frac{t - t_1}{2t_1}}, & t_1 \leqslant t \leqslant t_2 \\ a_0 e^{\chi}(t)^{1/2} - t_2 + t_1, & t \geqslant t_2 \end{cases} \tag{11}$$

and $a_0 = \text{const}$, t_1 is the time when vacuum energy starts to play a dominant dynamical role, and t_2 is the moment of reheating. We assume that $\chi = t_2 - t_1/2t_1 \simeq 60$, so during the de Sitter phase the scale factor $a(t)$ increases by a factor $\simeq 10^{26}$.

In the synchronous gauge ($\delta g_{00} = \delta g_{0i}$) the purely gravitational perturbations of the metric (10) $\delta g_{ik} = h_{ik}$ ($i, k = 1,2,3$) satisfy the equation

$$h_i''^k + 2\frac{a'}{a}h_i'^k + a^2 g^{lm} h_{i,l,m}^k = 0 \tag{12}$$

where prime denotes the derivative with respect to $\tau = \int dt/a(t)$, and a comma denotes the derivative with respect to the Cartesian spatial coordinates. Lifshitz (1946) has shown that the metric perturbations corresponding to the purely gravitational perturbations can be decomposed into tensor harmonics Q_{ik} defined as solutions of the following set of equations:

$$\Delta Q_{ik} = -\frac{n^2}{a^2}Q_{ik}, \qquad Q_i^i = 0, \qquad Q_{i,k}^k = 0 \tag{13}$$

where Δ denotes the three-dimensional Laplacian. Expanding h_{ik} into tensor harmonics $h_{ik} = \Sigma \nu_n(\tau) Q_{ik}(x)$, we obtain

$$\nu'' + 2\frac{a'}{a}\nu' + n^2\nu = 0 \tag{14}$$

where we have omitted the subscript n. Finally, introducing $\nu = \mu/a$ we can rewrite equation (14) in the form

$$\mu'' + \mu(n^2 - a''/a) = 0 \tag{15}$$

In the radiation-dominated era $a \approx \tau$ and $a'' = 0$, so at the very early stage and after reheating we have

$$\mu_{in} = A\sin(n\tau + \phi)$$
$$\mu_f = B\sin(n\tau + \psi) \tag{16}$$

where A, B, ϕ, and ψ are constants.

During the de Sitter phase equation (15) assumes the form

$$\mu'' + \mu\left[n^2 - \frac{2}{(2\tau_1 - \tau)^2}\right] = 0 \tag{17}$$

The general solution of this equation is

$$\mu = C_1\left[\sin n(2\tau_1 - \tau) + \frac{\cos n(2\tau_1 - \tau)}{n(2\tau_1 - \tau)}\right]$$
$$C_2\left[\cos n(2\tau_1 - \tau) - \frac{\sin n(2\tau_1 - \tau)}{n(2\tau_1 - \tau)}\right] \tag{18}$$

Assuming that μ and its first derivative are continuous and averaging over the initial phase ϕ it is possible to relate the final amplitude B with the initial amplitude A. The general formula is complicated so let us consider only some interesting limiting cases. For perturbations, which at the moment of the first transition are of moderately high frequency, such that $n\tau_1 \gg 1$, but $n\tau_1 e^{-\chi} \ll 1$ we obtain $\langle B^2/A^2\rangle \simeq (1/2)(e^{\chi}/n\tau_1)^4 \gg 1$. This means that in our model the amplitude of moderately high-frequency gravitational perturbation is very effectively amplified.

The high-frequency gravitational perturbations could be interpreted in terms of gravitons. The number density and energy density of gravitons of mode n is given by

$$N_n \sim \frac{(\text{amplitude})^2 n}{a^3}, \qquad \varepsilon_n \sim \frac{(\text{amplitude})^2 n^2}{a^4} \tag{19}$$

It is interesting to compare the number density and energy density of gravitons before and after the de Sitter phase, we have

$$N(\tau_2) = \frac{1}{2}N(\tau_1)\frac{e^{\chi}}{(n\tau_1)^4}, \qquad \varepsilon(\tau_2) = \frac{1}{2}\varepsilon(\tau_1)\frac{1}{(n\tau_1)^4} \tag{20}$$

The number density of moderately high-frequency gravitons increases but their energy density decreases. This is due to the fact that the frequency of gravitons is very strongly red shifted during the de Sitter phase and $\omega(\tau_2) = \omega(\tau_1)e^{-\chi}$.

The change in the number density and energy density of gravitons is even more apparent if we compare the number density and energy density of gravitons after the de Sitter phase with corresponding quantities calculated using adiabatic expansion law. From the adiabatic expansion law it follows that

$$N_n^A(\tau_2) = N_n(\tau_1)\frac{a^3(\tau_1)}{a^3(\tau_2)} = N_n(\tau_1)e^{-3\chi} \tag{21}$$

and

$$\varepsilon_n{}^A(\tau_2) = \varepsilon_n(\tau_1)\frac{a^4(\tau_1)}{a^4(\tau_2)} = \varepsilon_n(\tau_1)e^{-4\chi} \tag{22}$$

The number density and energy density of gravitons with moderately high frequency after the de Sitter phase is much larger than expected from the law of adiabatic expansion. This can be interpreted as classical description of the quantum process of graviton creation. The fact that gravitons can be created in homogeneous and isotropic Friedman model was first noticed by Grishchuk (1975).

The very-high-frequency gravitational perturbations for which $n\tau_1 e^{-\chi} \gg 1$ are not amplified and for such perturbations $\langle B^2/A^2 \rangle = 1$. The number density and energy density of very-high-frequency gravitons will therefore be adiabatically diluted. After the de Sitter phase the number density of very-high-frequency gravitons will be negligibly small and at the present epoch practically unobservable.

For low-frequency (long-wavelength) initial gravitational perturbations with n such that $n\tau_1 \ll 1$ we obtain $\langle B^2/A^2 \rangle \simeq (2/9)[e^{4\chi}/(n\tau_1)^2]$. The low-frequency gravitational perturbations are also very effectively amplified. However, the long-wavelength gravitational perturbations cannot be interpreted in terms of gravitons.

If the purely gravitational perturbations are generated by quantum vacuum fluctuations then their spectrum is described by

$$A_n{}^2 \simeq n^2 \tag{23}$$

In the model, which we are considering, the spectrum of very-high-frequency gravitational perturbations will not change, and $B_n{}^2 \simeq n^2$ for $n\tau_1 \gg e^{\chi}$. The spectrum of moderately high-frequency perturbations with $1 \ll n\tau_1 \ll e^{\chi}$ will change, and $B_n{}^2 \simeq n^{-2}$. The spectrum of low-frequency perturbations with $n\tau_1 \ll 1$ will become scale independent, with $B_n{}^2$ independent of n. This is the characteristic feature of the constant curvature perturbations.

We conclude therefore that if the inflationary phase occurs after the quantum epoch, the effects of quantum gravity could have been completely washed out, and present structure of the universe might depend only on physical processes which took place during and after the inflationary epoch.

I think that a word of caution is appropriate here. The inflationary model is very attractive but it is facing serious problems. To obtain desirable properties of the Higgs field effective potential one has to fine-tune the parameters. If the transition to the true vacuum state is not synchronous, density perturbations much larger than allowed by present observational limits could be created. This problem has been recently discussed by many authors; see for example comprehensive study by Bardeen, Steinhardt, and Turner (1983).

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